FRQ 6)

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$$f(x) = \begin{cases} \sqrt{x+1}, & x \text{ in } [0,3] \\ 5-x, & x \text{ in } (3,5] \end{cases}$$

a) continuous at $x=3$?

1) $f(3) = \sqrt{4} = 2$. it exists

2) $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \sqrt{x+1} = \sqrt{4} = 2$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 5-x = 5-3 = 2$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$, so $\lim_{x \to 3}$ exists

3) $f(3) = 2 = \lim_{x \to 3} f(x)$, so $f(x)$ is continuous at 3

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6b) find the average value of $f(x)$ on $[0,5]$

average value of $f(x)$ on $[0,5]$: $= \frac{1}{5-0}\int_0^5 f(x)\,dx$

$$= \frac{1}{5}\left[\int_0^3 \sqrt{x+1}\,dx + \int_3^5 5-x\,dx\right]$$

$$= \frac{1}{5}\left[\frac{2(x+1)^{3/2}}{3}\Big|_0^3 + \left(5x-\frac{x^2}{2}\right)\Big|_3^5\right]$$

$$= \frac{1}{5}\left[\frac{2(3+1)^{3/2}}{3} - \frac{2}{3}(0+1)^{3/2} + \left(5\cdot 5-\frac{5^2}{2}\right)-\left(5\cdot 3-\frac{3^2}{2}\right)\right]$$

$$= \frac{1}{5}\left[\frac{16}{3}-\frac{2}{3}+\frac{25}{2}-\frac{21}{2}\right]$$

$$= \frac{1}{5}\left[\frac{14}{3}+\frac{6}{3}\right] = \frac{4}{3}$$

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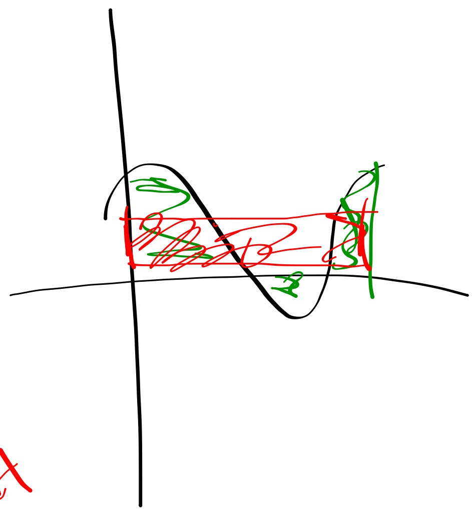
Net area between $f(x)$ & x-axis between a & b $= \int_a^b f(x)\,dx$



MVT for Integrals = there is a c on $[a,b]$ with $f(c)\cdot(b-a) = \int_a^b f(x)\,dx$

Average Value of $f(x)$ on $[a,b]$ = height of that rectangle

$= f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

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c) suppose we have a New function

$$g(x) = \begin{cases} k\sqrt{x+1}, & x \text{ in } [0,3] \\ mx+2, & x \text{ in } (3,5] \end{cases}$$

g is differentiable at $x=3$.

$\Rightarrow$ g is continuous at $x=3$ also.

a) continuity requires $\lim_{x\to 3} k\sqrt{x+1} = \lim_{x\to 3} mx+2$

so $2k = 3m+2$

b) $g'(x) = \begin{cases} \frac{d}{dx}(k\sqrt{x+1}) = \frac{k}{2\sqrt{x+1}}, & x \text{ in } (0,3) \\ \frac{d}{dx}(mx+2) = m, & x \text{ in } (3,5) \end{cases}$

c) differentiable requires

$\lim_{x\to 3} \frac{k}{2\sqrt{x+1}} = \lim_{x\to 3} m$

so $\frac{k}{4} = m \Rightarrow \boxed{k=4m}$

d) $k=4m \Rightarrow 2(4m) = 3m+2$

$8m = 3m+2$

$5m = 2$

$m = \frac{2}{5} \Rightarrow k = 4\left(\frac{2}{5}\right) = \frac{8}{5}$

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Part A, #26

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What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at $(3,2)$

implicit differentiation

undefined slope $\frac{dy}{dx} = \frac{\text{not zero}}{\text{zero}}$

Part A #27

Let $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

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$$\int x\,dx = \begin{cases} \frac{x^2}{2}+1 \\ \vdots \\ \frac{x^2}{2}+2 \\ \vdots \\ \frac{x^2}{2}+3 \\ \vdots \end{cases}$$